A Personal and Historical View of Computational Mathematics

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Math-IMS, CUHK,

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What is CM?

- Wiki: "*involves mathematical research in areas of science where computing plays a central and essential role, emphasizing algorithms, numerical methods, and symbolic methods.*"

- Newton, Gauss, Richardson, Von Neumann,..

Modern History

- CM modern history after WWII
  - Inst of Numer Analysis at UCLA 1947-1954
- SIAM founded April 1952
  - Main professional society promoting CAM
  - Journals: SINUM, SISC, MMS, SIIMS (AMS Comp Math)
  - Conferences, SIAGs, reports
- Algorithms "founded" circa 1952
  - SOR, CG, ADI, Simplex

CM & TC

- My UG/Grad CAM:
  - WKB, singular perturbation, asymptotics
  - Laplace/Fourier transforms, separation of variables, tensor analysis, game theory
  - NA from Issacson & Keller, Conte & De Boor, Dahlquist & Bjorck (GVL being written); NODE from Gear, NPDE from Mitchell, K&O; CS algs; some CFD
  - Hardware: PDP 11, IBM 360 (cards!), HP35C, HP2000, arpanet
  - Software: JCL, Fortran, Basic, APL, ALGOL
  - Algorithms: Least sq, SVD, fast Poisson solvers, quasi-newton methods, ALM
- I have worked on/participated in:
  - NPDE, NLA, MG, DD, CG, //, IP, Brain mapping, VLSI
  - Competition between finite difference vs finite elements for PDEs
  - NLA: Matrix factorizations, LS, QR, SVD, Lanczos, CG (GMRES etc), on Householder Symp Comm 1993-2005.
  - MG: From Brandt 78 MLAT paper on, MG for bifurcation probs, unstructured grids; EMG confs
  - DD: Chaired DD2 at UCLA 1988, on DDM Committee until DD12.
  - //: main frames, vector machines (Alliant), parallel machines (CM, Cosmic Cube, Ncube), GPU
  - Software: EISPACK, LINPACK, Matlab (before MathWork), Mathematica, Petsc
  - Applications:
    - CFD: MG/DD for unstructured meshes;
    - IP: TV, level set, L1, Compressed sensing
    - Brain mapping: Ricci flow, Yamabe eq, conformal mapping, Beltrami coef
    - VLSI opt: new paradigm, widely adopted in industry, hypergraphs
- Math I should have learned in school:
  - nonlinear analysis, differential geometry, stochasticity, BV, L1, Baysean

TOP 10 Sites for June 2016

For more information about the sites and systems in the list, click on the links or view the complete list.

Rank	Site	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	National Supercomputing Center in Wuxi China	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway NRCPC	10,649,600	93,014.6	125,435.9	15,371
2	National Super Computer Center in Guangzhou China	**Tianhe-2 (MilkyWay-2)** - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200GHz, TH Express-2, Intel Xeon Phi 31S1P NUDT	3,120,000	33,862.7	54,902.4	17,808
3	DOE/SC/Oak Ridge National Laboratory United States	**Titan** - Cray XK7 , Opteron 6274 16C 2.200GHz, Cray Gemini interconnect, NVIDIA K20x Cray Inc.	560,640	17,590.0	27,112.5	8,209
4	DOE/NNSA/LLNL United States	**Sequoia** - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom IBM	1,572,864	17,173.2	20,132.7	7,890
5	RIKEN Advanced Institute for Computational Science (AICS) Japan	K computer, SPARC64 VIIIfx 2.0GHz, Tofu interconnect Fujitsu	705,024	10,510.0	11,280.4	12,660
6	DOE/SC/Argonne National Laboratory United States	**Mira** - BlueGene/Q, Power BQC 16C 1.60GHz, Custom IBM	786,432	8,586.6	10,066.3	3,945
7	DOE/NNSA/LANL/SNL United States	**Trinity** - Cray XC40, Xeon E5-2698v3 16C 2.3GHz, Aries interconnect	301,056	8,100.9	11,078.9	

Rank	Site	System	Cores	Rmax (TFlop/s)	Rpeak (TFlop/s)	Power (kW)
1	RIKEN Center for Computational Science Japan	**Supercomputer Fugaku** - Supercomputer Fugaku, A64FX 48C 2.2GHz, Tofu interconnect D Fujitsu	7,630,848	442,010.0	537,212.0	29,899
2	DOE/SC/Oak Ridge National Laboratory United States	**Summit** - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband IBM	2,414,592	148,600.0	200,794.9	10,096
3	DOE/NNSA/LLNL United States	**Sierra** - IBM Power System AC922, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband IBM / NVIDIA / Mellanox	1,572,480	94,640.0	125,712.0	7,438
4	National Supercomputing Center in Wuxi China	**Sunway TaihuLight** - Sunway MPP, Sunway SW26010 260C 1.45GHz, Sunway NRCPC	10,649,600	93,014.6	125,435.9	15,371
5	NVIDIA Corporation United States	**Selene** - NVIDIA DGX A100, AMD EPYC 7742 64C 2.25GHz, NVIDIA A100, Mellanox HDR Infiniband	555,520	63,460.0	79,215.0	2,646

5	NVIDIA Corporation United States	**Selene** - NVIDIA DGX A100, AMD EPYC 7742 64C 2.25GHz, NVIDIA A100, Mellanox HDR Infiniband Nvidia	555,520	63,460.0	79,215.0	2,646
6	National Super Computer Center in Guangzhou China	**Tianhe-2A** - TH-IVB-FEP Cluster, Intel Xeon E5-2692v2 12C 2.2GHz, TH Express-2, Matrix-2000 NUDT	4,981,760	61,444.5	100,678.7	18,482
7	Forschungszentrum Juelich (FZJ) Germany	**JUWELS Booster Module** - Bull Sequana XH2000 , AMD EPYC 7402 24C 2.8GHz, NVIDIA A100, Mellanox HDR InfiniBand/ParTec ParaStation ClusterSuite Atos	449,280	44,120.0	70,980.0	1,764
8	Eni S.p.A. Italy	**HPC5** - PowerEdge C4140, Xeon Gold 6252 24C 2.1GHz, NVIDIA Tesla V100, Mellanox HDR Infiniband Dell EMC	669,760	35,450.0	51,720.8	2,252
9	Texas Advanced Computing Center/Univ. of Texas United States	**Frontera** - Dell C6420, Xeon Platinum 8280 28C 2.7GHz, Mellanox InfiniBand HDR Dell EMC	448,448	23,516.4	38,745.9	
10	Saudi Aramco Saudi Arabia	**Dammam-7** - Cray CS-Storm,	672,520	22,400.0	55,423.6	

CSE2000 Top 10 Algorithms

- Monte Carlo
- Simplex
- Krylov
- Matrix decomposition
- Fortran optimizing compiler
- Francis QR
- Quicksort
- FFT
- Integer relation detection
- Fast multipole method

- No update as of 1/28/2021

Other New Algorithmic Paradigms

- Multigrid
- DD
- Multiscale
- Uncertainty
- Continuous vs discrete: Diff Equ vs graphs

- 2021: Deep Neural Networks?

Criteria

- Speed (flops, comp complexity, time)
- Accuracy
- Scalability
- Parallelism, communication/latency
- Memory efficiency
- Impact (internal and external)

Top 10 Most Cited Recent CM Papers (TC 04/30/21)

Ref: Golub-Van Loan (4th ed 2012): 52717/72912 citations! Metropolis et al (1953, J Chem Phys) 31807 citations. Hestenes-Stiefel Conjugate Gradient (1952 JBS) 6379 citations.

2014 Dec

1. 1(15794/?) Donoho, Compressed Sensing, IEEE TIT 2006
2. 2(12242/17463) Osher-Sethian, level set, JCP 1988
3. 3(10162/?) Donoho, Soft Thresholding, IEEE TIT 1995
4. 6(10157/17084) Candes-Romberg-Tao, Compr Sensing, IEEE TIT 2006
5. 8(9281/15826) Rudin-Osher-Fatemi, TV denoising, Physica D 1992
6. 5(9576/13061) Saad-Schultz, GMRES, SISC 1986
7. 9(8223/12715) Chan-Vese, Segmentation, IEEE TIP 2001
8. 4(9370/11937) Daubechies, Ortho Basis for Wavelets, CPM 1984
9. 7(8795/?) Donoho-Johnstone, wavelet shrinkage, Biometrika 1994
10. 10(4296/5803) Greengard-Rokhlin, Fast Multipole, JCP 1987

11. (3772/?) Brandt, Multigrid, Math Comp 1977

Most citations above are not by "peers" but by "users".

LeCun, Bengio, Hinton "Deep Learning", Nature 2015. 38352 citations!

Hinton "Image classification with Deep Convolution Neural Networks", 2012. 85306 cites!

Computational Linear Algebra

- EISPACK, LINPACK, LAPACK, Matlab
- Matrix factorizations: QR, SVD, RRQR,
- Tensors
- Least sqs: LS, TLS
- Sparsity: GE, ordering, fill-ins
- Iterative meths: CG, Lanczos, GMRES, preconditioners
- Revived twice by: parallelism, search/Google, completion/Netflix

Multigrid/Domain Decompositon

- A paradigm shift
- Russian influence
  - motivated by efficient use of limited computer power
- MLAT – Brandt
- DD – motivated by parallel computers
- Subspace decomposition & correction (Xu)
- Fast Multipole Method – particle simulation

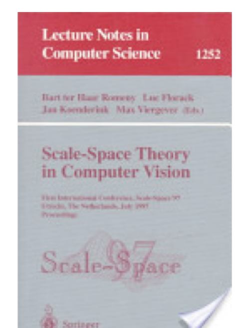
CFD to IP

- FD has provided a challenging testbed for CM for several decades
- Multi-dim, multi-component, mutli-scale, nonlinear, discontinuities, complex geometry/mesh, time-dep,
- IP is “new CFD”: all of above (except reg mesh)

Imaging science

- Relatively new to Math Community
- SIIMS founded 2007.
- Imaging journals:
  SIAM SIIMS
  IEEE TIP
  JMIV
  IJCV

- Imaging conferences:
  SIAM Imaging conference
  Scale-Space conference
  IEEE ICIP
- The new “CFD”. Part of “ Data S



Report of the SIAM Journal Committee on **"Proposal for a New SIAM Journal on Imaging Sciences"**, submitted by Tony Chan and Kirk Jordan

Margaret Cheney
Edward Olmstead
Mac Hyman
Tim Kelley (SIAM Vice-President for Publications)
Tom Manteuffel
Nick Trefethen
Benjamin White
David Williamson
Margaret Wright (chair)

July 2, 2006

Main issue is "topic" journal. NOT Imaging as a topic

Forward or backward looking?

ACM, IEEE having same discussion or concern?

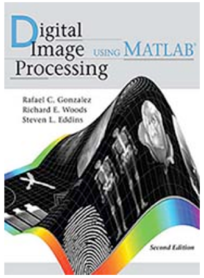
1 Committee Charge; Review Process

Following an extended discussion at the July 2005 SIAM annual meeting in New Orleans, the SIAM Council requested that Tony Chan and Kirk Jordan prepare a proposal for a new SIAM journal in imaging sciences. The proposal was received by SIAM on April 20, 2006. The SIAM Journal Committee was then asked by the Vice-President for Publications, Tim Kelley, to review the proposal and prepare a report. This report summarizes that review and will be presented to the SIAM Council in Boston on July 13, 2006.

In conducting its review of the proposal, each member of the Journal Committee was asked by Margaret Wright (its chair) to send comments on the proposal. Her email of April 21, 2006, to the committee said:

> It is now our job to make a recommendation to the SIAM Council (which will meet in mid-July at the annual meeting) as to whether SIAM should establish such a journal.

What Math brings to IP

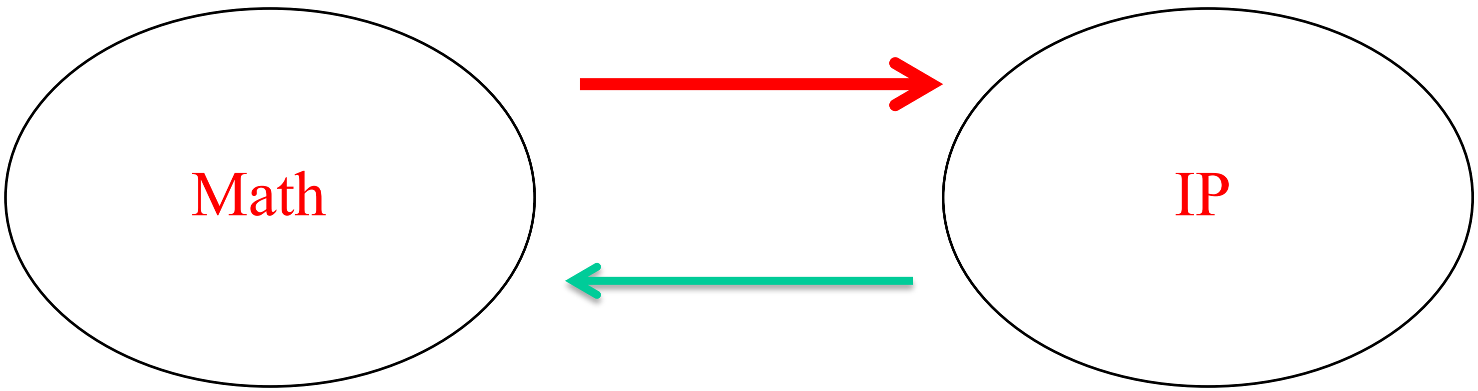


IEEE TRANSACTIONS ON
IMAGE PROCESSING
A PUBLICATION OF THE IEEE SIGNAL PROCESSING SOCIETY

Volume 7, Number 3, March 1998

Image Processing and Applied Maths

PDEs (CFD, viscosity solution)
Calculus of variations
Differential geometry (level set method)
Smooth & non-smooth functional analysis
(wavelets/Besov, total variation/BV)



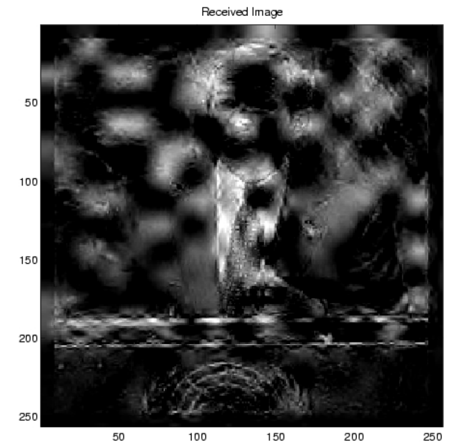
Compressed sensing, L1 optimization (Bregman)
Convexification for geometric problems
Relaxation for graph optimization problems

Compressed Sensing
Candes, Donoho, Romberg, Tao 2006

- Sparsity: L0
- Convexity: L1
- Randomness: Restricted Isometry Property
- Exact recovery
- IPAM origin
- TV Wavelet Inpainting connection

TV Wavelet inpainting [Chan, Shen & Zhou '04]

Can you recognize this person?



Original image downloaded from internet
Damaged image: 50% of wavelet coefficients
(including low frequencies) are randomly lost.

- Model I (for noise-less images):

$$\min_{\beta_{j,k}} \int |\nabla u| \;\; \text{s.t.} \;\; u = \sum_{j,k} \beta_{j,k} \underbrace{\Phi_{j,k}}_{\text{wavelets}}, \;\; \beta_{j,k} = \beta^0_{j,k} \quad \text{(observed coefs)}$$

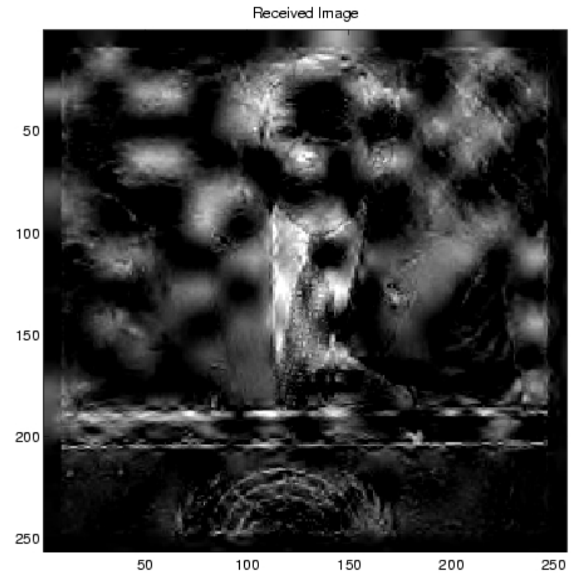
No parameters. Problem dimension << #coefs.

- Model II (for noisy images):

$$\min_{\beta_{j,k}} \int |\nabla u| + \lambda_I \|u - u_0\|^2 \;\; (u_0 \text{ observed image})$$

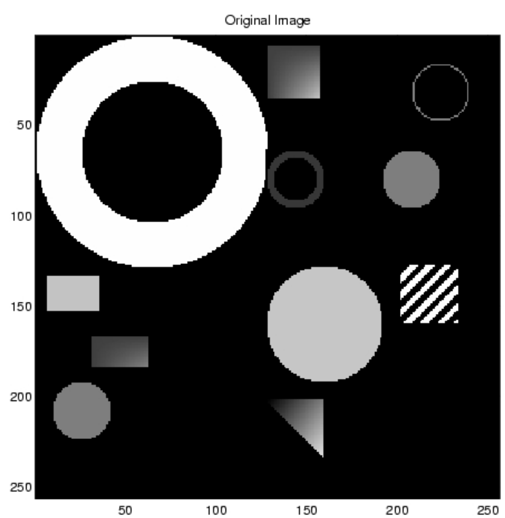
Restoration result

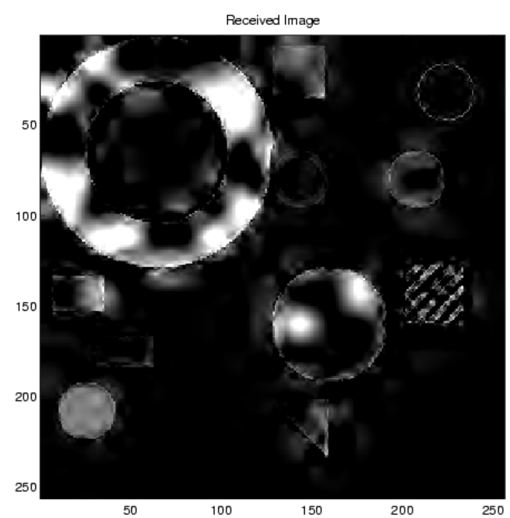
Original image

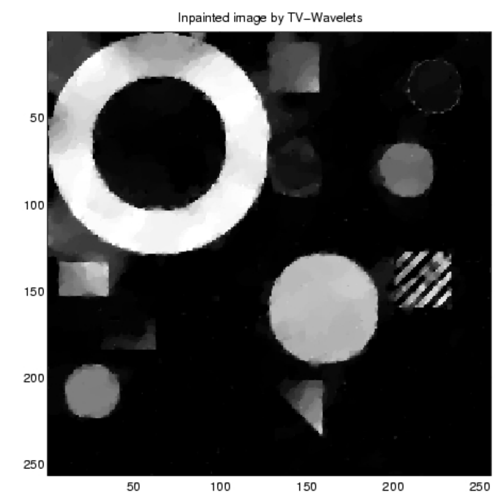


Damaged image, 50% of wavelet coefficients are randomly lost. PSNR = 10.9

Model II: Recovered image, keeping undamaged coefficients unchanged. PSNR 18.8.

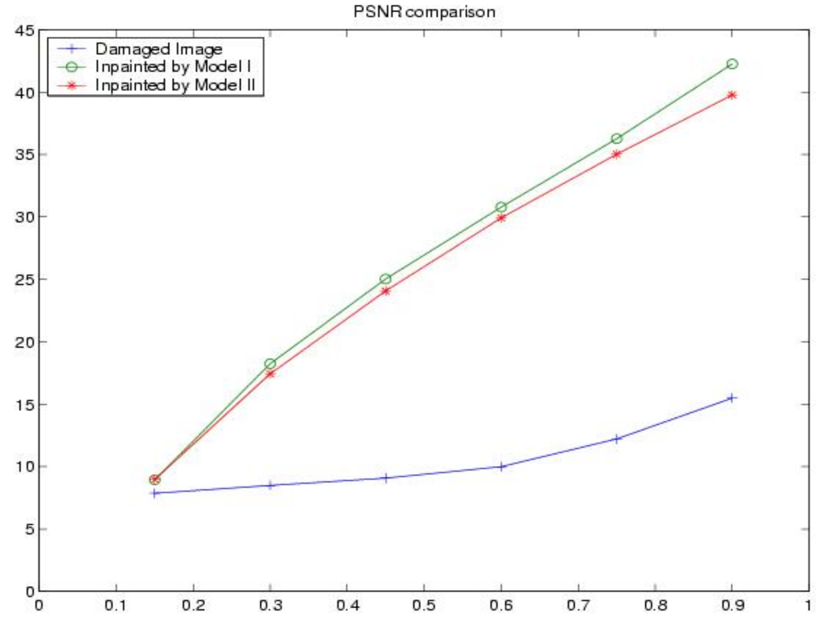






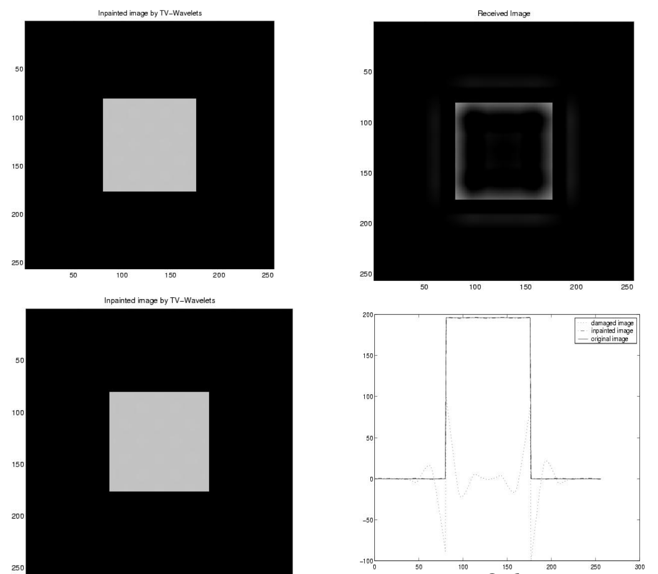
PSNR vs %Coefs Retained

X-axis: percentage of randomly retained wavelet coefficients
Y-axis: improvements in PSNR after inpainted by Model I (green) and Model II (red). As less coefficients are missed, the improvement in PSNR becomes larger.



An Extreme Example

Upper left: Original clear square.
Upper right: Lost all but one nonzero coefficients in the low-low frequency subband, while keep all high frequencies (PSNR=11.2dB).
Lower left: Recovered by Model I. Perfect reconstruction (PSNR=61dB).
Lower Right: Cross-section at x=128. The inpainted profile is visually indistinguishable from the original.



A preview of Compressed Sensing!

Compressed Sensing in IP
[Candes, Romberg & Tao '06, Donoho '06]

Reconstruct the signal u in R^n from m measurements f with m<<n :

$$f = Au, \;\; u \in \mathbb{R}^n, \;\; f \in \mathbb{R}^m, \;\; A \in \mathbb{R}^{n\times m}$$ (system is underdetermined)

Assume u is sparse then the problem is: $\min_u \;\|\Psi u\|_0 \;\, s.t. \;\, f = Au$

Theorem: For m>=c.log n, the signal u can be reconstructed exactly (under reasonable assumptions) solving the problem:

$$\min_u \;\|\Psi u\|_1 \;\, s.t. \;\, f = Au$$ L1 relaxation (tight)

For $\|\Psi u\|_1 = TV(u) = \int |\nabla u|$ and $A = R\Phi \begin{cases} \Phi\text{: sensing basis (wavelet, Fourier)} \\ \text{R: measurement extractor} \end{cases}$

=> The CS problem is equivalent to the TV wavelet inpaiting problem.

Algorithm Connections, Old and New

Quadratic Penalty	$\longleftrightarrow$	Penalty method (Reformulates total variation penalty as constrained optimization problem) [Wang, Yin, Zhang 2007]
Uzawa's Method	$\longleftrightarrow$	Linearized Bregman (Efficient for large scale problems) [Darbon, Osher, Yin, Goldfarb 2007]
Proximal Forward Backward Splitting	$\longleftrightarrow$	Fixed Point Continuation (FPC) and many related iterative thresholding methods for L1 minimization [Hale, Yin, Zhang 2007]
Arrow-Hurwicz	$\longleftrightarrow$	Primal Dual Hybrid Gradient (PDHG) (Excellent for TV denoising) [Zhu, Chan 2008]
Preconditioned Method of Multipliers/ Proximal Method of Multipliers	$\longleftrightarrow$	Bregman Operator Splitting (BOS) (Proposed for nonlocal TV; very generally applicable) [Zhang, Burger, Bresson, Osher 2009]
Preconditioned ADMM (Linearization of penalty also related to surrogate function and optimization transfer)	$\longleftrightarrow$	Split Inexact Uzawa, Modified PDHG, Chambolle/Pock Method, He/Yuan Variants... (Many versions of this very versatile method) [Zhang, Burger, Osher, Esser, Chan, Chambolle, Pock...2009]
Newton-like Methods	$\longleftrightarrow$	CGM [Chan, Golub & Mulet '95] Semismooth Newton for TV [Hintermuller, Stadler] (Uses second order information, can be superlinearly convergent)

Lessons

1. Right model more important than fast algorithm
2. Simplicity trumps everything else
3. Proof is good (but not absolutely necessary) but good idea is a must
4. Old methods can find new use: IP opt
5. New applications can inspire new CM: IP, search
6. Hardwares change, good ideas last: Hypercubes, Connection Machine
7. Ideas for one problem can be useful for another
    1. TVD for shock capturing -> TV for IP
8. What appears different may be related
    1. IP: PDE vs graph modelling & algorithms
9. New Math plus old tricks - differential geometry + ALM
10. Be the first to spot a math idea or a rich application

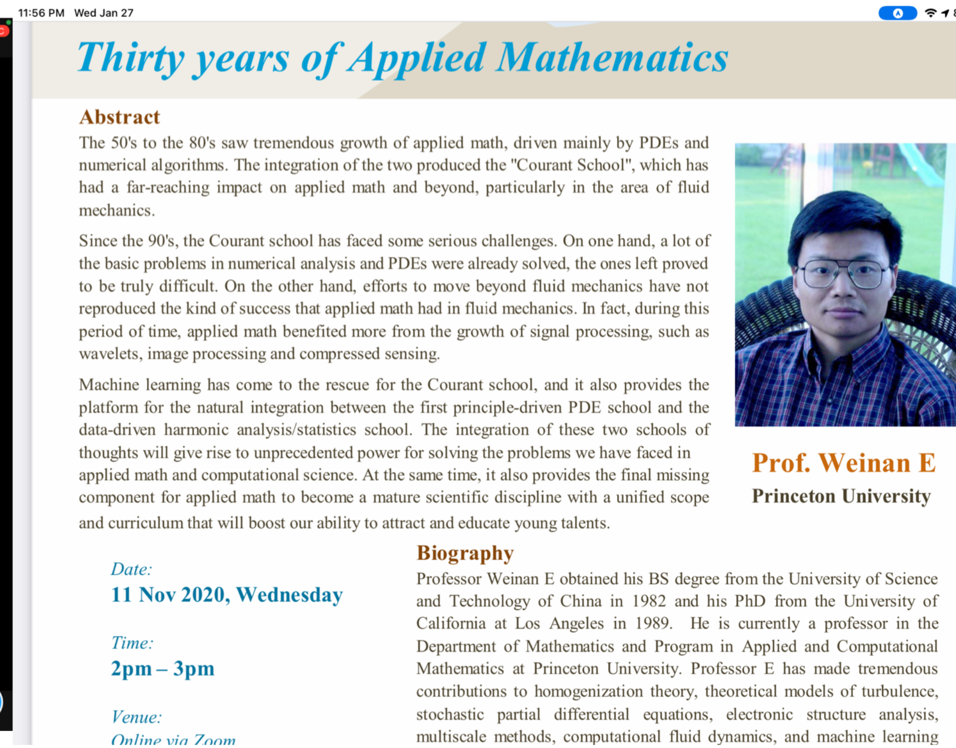
Future

- Plenty of challenges in basic S&E

- Biology (genetics, brain, diseases,….)
- Big data (mining, learning,…)
- Uncertainty (Quantization, randomization…)
- IP to Vision, Comp Photography,..

- New math; New applications
- 2021: Machine Learning – the new CFD?

Back to the Future?



11:56 PM Wed Jan 27

Thirty years of Applied Mathematics

Abstract

The 50's to the 80's saw tremendous growth of applied math, driven mainly by PDEs and numerical algorithms. The integration of the two produced the "Courant School", which has had a far-reaching impact on applied math and beyond, particularly in the area of fluid mechanics.

Since the 90's, the Courant school has faced some serious challenges. On one hand, a lot of the basic problems in numerical analysis and PDEs were already solved, the ones left proved to be truly difficult. On the other hand, efforts to move beyond fluid mechanics have not reproduced the kind of success that applied math had in fluid mechanics. In fact, during this period of time, applied math benefited more from the growth of signal processing, such as wavelets, image processing and compressed sensing.

Machine learning has come to the rescue for the Courant school, and it also provides the platform for the natural integration between the first principle-driven PDE school and the data-driven harmonic analysis/statistics school. The integration of these two schools of thoughts will give rise to unprecedented power for solving the problems we have faced in applied math and computational science. At the same time, it also provides the final missing component for applied math to become a mature scientific discipline with a unified scope and curriculum that will boost our ability to attract and educate young talents.

Prof. Weinan E
Princeton University

Date:
11 Nov 2020, Wednesday

Time:
2pm – 3pm

Venue:
Online via Zoom

Biography

Professor Weinan E obtained his BS degree from the University of Science and Technology of China in 1982 and his PhD from the University of California at Los Angeles in 1989. He is currently a professor in the Department of Mathematics and Program in Applied and Computational Mathematics at Princeton University. Professor E has made tremendous contributions to homogenization theory, theoretical models of turbulence, stochastic partial differential equations, electronic structure analysis, multiscale methods, computational fluid dynamics, and machine learning

Carnegie Initiative on the Doctorate

In 2001 The Carnegie Foundation for the Advancement of Teaching launched a multiyear project to examine the doctoral degree in the United States. Entitled the Carnegie Initiative on the Doctorate (CID), the project aims to stimulate rethinking and renewal of doctoral programs in six disciplines, one of which is mathematics. During 2003 the CID will publish a set of twelve essays, two in each discipline, about doctoral education. And earlier this year, thirty-two "partner departments", including eight mathematics departments, were chosen as participants in the initiative.

Founded by Andrew Carnegie in 1905, the Carnegie Foundation has a long tradition of carrying out research and policy studies in education. One of its most famous and influential projects was the 1910 "Flexner report", a tough indictment of medical schools in the United States that led to widespread reform. It was partly because of his fame as author of that report that Abraham Flexner became the founding director of the Institute for Advanced Study. Although the Flexner report is sometimes mentioned in connection with the CID, the similarity is not strong. "Of course we would like [the CID] to be influential," remarked Chris Golde, research director for the CID. The difference is that Flexner clearly had an agenda in mind when he wrote his report, while the CID does not. "We are committed to that [agenda] needing to arise from the disciplines," Golde said.

In fact, the CID does not even take it for granted that doctoral education needs a huge overhaul. "I don't see [the CID] as necessarily dramatically changing what we do," said Kevin Corlette, chair of the University of Chicago mathematics department, one of the CID participating departments. "The idea is to go back to basic questions about what we are trying to accomplish in doctoral education and to think about what we could do differently or better." A sign of the excellence of U.S. doctoral programs is that they attract students from all over the globe. Still, evolution in the societal and academic contexts in which these programs exist has sometimes caused mismatches between the goals of doctoral programs and what is expected of the programs' graduates. The idea of the CID is to ask anew the question, What is the purpose of doctoral education today? George Walker, a theoretical physicist at Indiana University who serves as the CID project director, said the main goal is to "convince people to think carefully and deeply about PhD programs, and then to act on their thoughts, just as they do in their disciplinary research."

An organizing notion for the initiative is that doctoral programs should produce "stewards of the discipline". In addition to having made an original contribution to research in the discipline, a steward possesses perspective on the history of the discipline, on the "big questions" and ideas that drive the field, and on its relations to other areas. A steward should also be a communicator in the broadest sense: someone who not only can teach students effectively but also can communicate in such a way that the tools and ideas of the discipline are available to those outside it. The notion of a steward of the discipline "calls in an ethical dimension," Golde remarked. "It's about integrity.

It's about, What are your responsibilities to the discipline?"

The CID is "conceptually based," Golde explained. "Its heart is in ideas, not in very specific practices." The purpose is not, say, to promote the use of interdisciplinary team projects in doctoral programs. Rather, the goal would be to get departments to think carefully about the communication skills that stewards of the discipline should possess. Departments would then consider what kinds of experiences—team projects may be one of them—would lead to the development of those skills. As Walker put it, "What experiences should future stewards of the field have in graduate school to allow them to evolve and be effective in a future we can't imagine, a future thirty or fifty years from now?"

Six fields were chosen for the CID: chemistry, education, English, history, mathematics, and neuroscience. There were several reasons for choosing these particular disciplines. One was the aim of including disciplines that span a wide area of academe. Carnegie also looked for central, fundamental fields where there are a large number of doctoral programs and students. The inclusion of neuroscience reflected the desire to bring in a multidisciplinary field.

In addition, Carnegie sought areas in which discussions of the doctorate were already taking place, and in that regard mathematics was a natural choice. Over the past decade or so the mathematical community has engaged in intensive discussions about the doctoral program. Among the major stimulants of these discussions were the reports *Educating Mathematical Scientists: Doctoral Study and the Postdoctoral Experience in the United States* (National Academy Press, 1992) and *Towards Excellence: Leading a Doctoral Mathematics Department in the 21st Century* (AMS, 1999). These discussions influenced the establishment of the VIGRE (Vertical Integration of Research and Education in the Mathematical Sciences) program of the National Science Foundation, which has stimulated much innovation and change in doctoral programs.

To begin the CID, Carnegie commissioned the writing of twelve essays, two in each of the six disciplines, that explore the purpose of doctoral education and the notion of "stewards" in the context of each discipline. The essays are not "how-to" manuals for structuring doctoral programs; their purpose is not to tell departments what to do but rather to provoke discussions within disciplines and across academe. The essays in mathematics were written by Hyman Bass of the University of Michigan, who is now past president of the AMS, and Tony Chan of the University of California, Los Angeles (the essays by Bass and Chan will be published in future issues of the *Notices*). All of the essays are scheduled to be completed in spring 2003 and will eventually be published together as a book.

CID Participating Mathematics Departments

Partner Departments

Duke University
Ohio State University
State University of New York, Stony Brook
University of Chicago
University of Illinois, Urbana-Champaign
University of Michigan, Ann Arbor
University of Nebraska, Lincoln
University of Southern California

Allied Departments

Howard University
Kent State University
University of North Carolina, Chapel Hill
University of Utah

The second part of the CID began in early 2003 with the selection of thirty-two "partner departments". These departments have made a commitment to undertake a thorough examination of their doctoral programs. The intention is that the commissioned essays will stimulate the departments' discussions. CID staff will visit the partner departments, not for the purpose of evaluating them, but rather to encourage discussions and bring in new ideas. There will also be periodic meetings of partner department representatives, the first one taking place in July 2003 at the Carnegie headquarters on the campus of Stanford University. After the initial conceptual phase, the departments will experiment with new ways of designing doctoral programs. In the "research and dissemination" stage, departments will evaluate the outcomes of their experiments and share them with colleagues at other institutions; the Carnegie Foundation will also help to publicize ideas and results growing out of the CID.

The departments were chosen after a call for applications was issued last fall. Because there were more worthy applications than it could accommodate, Carnegie also selected some "allied departments" that can participate in discussions and receive materials but which will not have site visits or support for attending CID meetings. (See the sidebar for a listing of partner and allied mathematics departments.)

The VIGRE program has had an enormous effect on mathematics departments across the country, spawning innovations in education at the undergraduate and graduate levels. One reason for the large impact is that the goals of VIGRE are backed by grant dollars. By contrast, the CID provides essentially no funding to its partner departments

The Mathematics Doctorate: A Time for Change?

Tony F. Chan

The Carnegie Foundation commissioned a collection of essays as part of the Carnegie Initiative on the Doctorate (CID). Essays and essayists represent six disciplines that are part of the CID: chemistry, education, English, history, mathematics, and neuroscience. Intended to engender conversation about the conceptual foundation of doctoral education, the essays are a starting point and not the last word in disciplinary discussions. Those faculty members, students, and administrators who work in the discipline are the primary among multiple audiences for each of these essays. © 2003 by the Carnegie Foundation for the Advancement of Teaching. Reprinted with permission.

Comments on the essays and on the CID are welcome and may be sent to `cid@carnegiefoundation.org`. Further information may be found at the website `http://www.carnegiefoundation.org/cid` and in the article "The Carnegie Initiative on the Doctorate", by Allyn Jackson, *Notices*, May 2003, pages 566–8.

The other Carnegie essay about mathematics, by Hyman Bass, appeared in the August 2003 issue of the *Notices*, pages 767–76.

—*Allyn Jackson*

One can argue, with ample evidence, that in U.S. universities the system of producing mathematics doctorates is doing very well and needs no major overhaul. It is widely recognized that however poor our K–12 mathematics education—and perhaps also our undergraduate mathematics education—might be, our graduate programs in mathematics are the best and the envy of the world. Top students from around the world are still beating on our doors to get into our doctoral programs. We train them well, and many of these students become international leaders in their research fields. Take, for example, the two 2002 Fields Medalists and the Nevalinna Prize winner. Even though the press (at least the press in Beijing, where I read the news) referred to them as French, Russian, and Indian, two received their doctoral training at U.S. universities. We also seem to be succeeding in getting new support from the federal government for mathematical sciences. The recent increase in funding for the NSF [National Science Foundation] specifically targets the Division of Mathematical Sciences, and doctoral training, in particular for U.S. students, is a core part of this new funding program. Even Hollywood seems to be working in our favor, in view of the generally positive image of mathematics generated by movies such as *A Beautiful Mind*.

However, there are many signs that not all is well with our doctoral programs. Top, talented students, especially those born in the U.S., are choosing fields other than mathematics for graduate study. Many mathematics departments, especially those outside the "top tier", are having trouble filling their graduate programs with reasonably prepared and talented students. As a field of science, mathematics is underfunded compared to other sciences. Most of our doctoral students are supported by teaching assistantships rather than by fellowships or research assistantships. Our doctoral students are taking too long to get their degrees, and they are not sufficiently and broadly trained for career paths outside of academia. Other scientists and academic administrators perceive us as an insular and, worse, irrelevant community.

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2. Ensuring That Mathematics Is Part of Science
3. Ensuring Societal Support for Mathematics Doctoral Education
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