MATH-IMS Joint Applied Mathematics Colloquium Series
The Chinese University of Hong Kong

This MATH-IMS Joint Colloquium Series is organized by Center for Mathematical Artificial Intelligence (CMAI), under Department of Mathematics and Institute of Mathematical Sciences (IMS) at The Chinese University of Hong Kong. The colloquium series focuses on mathematics and applications of artificial intelligence, big data and related topics.

Date: May 5, 2023 (Friday)
Time: 9:00-10:00 (Hong Kong Time)
Zoom Link: https://cuhk.zoom.us/j/92775210812

A Fast Butterfly-compressed Hadamard-Babich Integrator for High-Frequency Helmholtz Equations in Inhomogeneous Media with Arbitrary Sources

Speaker: Professor Jianliang Qian,
Department of Mathematics and Department of CMSE, Michigan State University

Abstract: We present a butterfly-compressed representation of the Hadamard-Babich (HB) ansatz for the Green's function of the high-frequency Helmholtz equation in smooth inhomogeneous media. For a computational domain discretized with N_v discretization cells, the proposed algorithm first solves and tabulates the phase and HB coefficients via eikonal and transport equations with observation points and point sources located at the Chebyshev nodes using a set of much coarser computation grids, and then butterfly compresses the resulting HB interactions from all N_v cell centers to each other. The overall CPU time and memory requirement scale as $O(N_v \log_2 N_v)$ for any bounded 2D domains with arbitrary excitation sources. A direct extension of this scheme to bounded 3D domains yields an $O(N_v^{4/3})$ CPU complexity, which can be further reduced to quasi-linear complexities with proposed remedies. The scheme can also efficiently handle scattering problems involving inclusions in inhomogeneous media. Although the current construction of our HB integrator does not accommodate caustics, the resulting HB integrator itself can be applied to certain sources, such as concave-shaped sources, to produce caustic effects. Compared to finite-difference frequency-domain (FDFD) methods, the proposed HB integrator is free of numerical dispersion and requires fewer discretization points per wavelength. As a result, it can solve wave-propagation problems well beyond the capability of existing solvers. Remarkably, the proposed scheme can accurately model wave propagation in 2D domains with 640 wavelengths per direction and in 3D domains with 54 wavelengths per direction on a state-the-art supercomputer at Lawrence Berkeley National Laboratory. This is a joint work with Dr. Yang Liu, Jian Song and Robert Burridge.

Bio: Prof. Jianliang Qian is currently serving as a professor with a joint appointment in the Department of Computational Mathematics, Science and Engineering and the Department of Mathematics at Michigan State University. Prior to joining MSU in 2007, he was an assistant professor at Wichita State University (2005–2007), and served as a CAM assistant professor at UCLA (2002–2005). Prof. Qian earned his Ph.D. in 2000 at Rice University, then was a postdoc fellow at the Institute of Mathematics and its Applications at the University of Minnesota (2000-2002). Prof. Qian has published about 70 journal papers, has been leading a big team and been the organizer of many academic events. The central theme of his research has been foundations and fast algorithms for computational geometrical optics and high-frequency waves, and applications to seismic imaging and medical imaging. His research program on computational geometrical optics and high-frequency waves has been continuously funded by NSF since 2005.